# Official symbols and nomenclature of The Society of Rheology

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## Official symbols and nomenclature of The Society of Rheology

The 12 tables that follow are the result of the hard work of the Ad Hoc Committee on Official Nomenclature and Symbols (John M. Dealy, Chair; Jeffrey Morris, Faith Morrison, and Dimitris Vlassopoulos) that was appointed by The Society of Rheology Executive Committee in 2012. The last major revision was done in 1984; this supersedes all prior versions, including the one published on pp. 253–265 of volume 39 of this journal in 1995. In the course of their work, the committee consulted numerous prominent rheologists working in specialized areas of rheology. These tables of Official Nomenclature and Symbols were approved for distribution here and online by The Society of Rheology Executive Committee in February 2013.

Ralph H. Colby, Editor

Name	Definition	Symbol	SI units <sup>a</sup>
Direction of flow (simple shear)	Figure 1	$x_1$	m
Displacement in the $x_1$ direction	Figure 1	$X_1$	m
Direction of velocity gradient (simple shear)	Figure 1	<i>x</i> <sub>2</sub>	m
Neutral direction (simple shear)		<i>x</i> <sub>3</sub>	m
Shear stress	F/A	σ	Pa
Shear strain	$dX_1/dx_2$	γ	
Shear rate	$dv_1/dx_2$	ý	$s^{-1}$
Viscosity	$\sigma/\dot{\gamma}$	$\eta(\dot{\gamma})$	Pa s
Yield stress		$\sigma_{ m y}$	Pa
Yield strain		γ <sub>y</sub>	
First normal stress difference	$\sigma_{11} - \sigma_{22}$	$N_1$	Pa
Second normal stress difference	$\sigma_{22} - \sigma_{33}$	$N_2$	Pa
First normal stress coefficient	$N_1/\dot{\gamma}^2$	$\Psi_1(\dot{\gamma})$	Pa s <sup>2</sup>
Second normal stress coefficient	$N_2/\dot{\gamma}^2$	$\Psi_2(\dot{\gamma})$	Pa s <sup>2</sup>
Normal stress ratio	$-N_1(\dot{\gamma})/N_2(\dot{\gamma})$	$\Psi(\dot{\gamma})$	
Zero-shear viscosity (limiting low shear rate viscosity)	$\eta \ (\dot{\gamma} \rightarrow 0)$	$\eta_0$	Pa s
Critical molecular weight for entanglement effect on $\eta_0$		$M_{\rm C}$	b
Limiting high shear rate viscosity	$\eta \; (\dot{\gamma} \to \infty)$	$\eta_{\infty}$	Pa s
Zero-shear first normal stress coefficient	$\Psi_1 \ (\dot{\gamma}  ightarrow 0)$	$\Psi_{1,0}$	Pa s <sup>2</sup>
Power law index	$\sigma = K  \dot{\gamma}^{n-1}  \dot{\gamma}$	n	_
Consistency	(see previous line)	Κ	Pa s <sup>n</sup>

**TABLE I.** Steady simple shear (viscometric flows).

<sup>a</sup>SI allows either a dot between units or a space, as used here.

<sup>b</sup>IUPAC recommends *molar mass* (MM), which has SI units of g/mol. But *molecular weight* (MW) is widely used, and ACS accepts both terms. However, MW is in fact a dimensionless ratio that is numerically very close to MM (g/mol), and one cannot "change its units." The number often called "molecular weight (kg/mol)" is actually MW/1000 (no units). This quantity can properly be called *molar mass* with units of kg/mol.

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FIG. 1. Simple shear.

#### TABLE II. Linear viscoelasticity.

Name	Definition	Symbol	Units
Simple shear			
Shear modulus of a solid	$\sigma/\gamma$	G	Pa
Relaxation modulus (shear)	$\sigma(t)/\gamma$	G(t)	Pa
Relaxation spectrum	a	$H(\tau)$	Pa
Memory function	-dG(s)/ds	<i>m</i> (s)	$Pa s^{-1}$
Creep compliance (shear)	$\gamma(t)/\sigma$	J(t)	$Pa^{-1}$
Equilibrium compliance of solid	$J(t)  (t \to \infty)$	$J_{\rm e}$	$Pa^{-1}$
Recoverable compliance	$J(t) - t/\eta_0$	$J_{\rm r}(t)$	$Pa^{-1}$
Steady-state compliance of fluid	$J(t) - t/\eta_0  (t \to \infty)$	$J_s^0$	$Pa^{-1}$
Molecular weight for entanglement effect on $J_s^0$	b	$M_{C}^{\prime}$	
Retardation spectrum	c	$L(\tau)$	$Pa^{-1}$
Small-amplitude oscillatory shear			
Strain amplitude	$\gamma(t) = \gamma_0 \sin \omega t$	γo	_
Loss angle (phase angle)	$\sigma(t) = \sigma_0 \sin(\omega t + \delta)$	δ	rad
Stress amplitude	$\sigma(t) = \sigma_0 \sin(\omega t + \delta)$	$\sigma_0$	Pa
Complex modulus	G' + iG''	$G^*$	Pa
Absolute magnitude of $G^*$	$\sigma_0/\gamma_0$	$ G^* $ or $G_d$	Pa
Storage modulus	$G_{ m d}\cos\delta$	G'	Pa
Loss modulus	$G_{\rm d} \sin \delta$	$G^{\prime\prime}$	Pa
Complex viscosity	$\eta' - i\eta''$	$\eta^*$	Pa s
Absolute magnitude of $\eta^*$	$\sigma_0/\omega\gamma_0$	$ \eta^* $	Pa s
Dynamic viscosity (in phase with strain rate)	$G''/\omega$	$\eta'$	Pa s
Out-of-phase (with strain rate) component of $\eta^*$	$G'/\omega$	$\eta''$	Pa s
Complex compliance	J' - iJ''	$J^*$	$Pa^{-1}$
Absolute magnitude of $J^*$	$\gamma_0/\sigma_0 = 1/G_d$	$ J^* $	$Pa^{-1}$
Storage compliance	$(\gamma_0/\sigma_0)\cos\delta$	J'	$Pa^{-1}$
Loss compliance	$(\gamma_0/\sigma_0)\sin\delta$	J''	$Pa^{-1}$
Plateau modulus	d	$G_{ m N}^0$	Pa
Tensile extension			
Net tensile stress	$\sigma_{ m zz} - \sigma_{ m rr}$	$\sigma_{ m E}$	Pa
Hencky strain	$\ln(L/L_0)$	3	_
Hencky strain rate	$d(\ln L)/dt$	ż	$s^{-1}$
Tensile relaxation modulus	$\sigma_{\rm E}(t)/\varepsilon_0$	E(t)	Pa
Tensile creep compliance	$\varepsilon_0(t)/\sigma_{\rm E}$	D(t)	$Pa^{-1}$

 ${}^{\mathrm{a}}G(t) = \int_{-\infty}^{\infty} H(\tau) [\exp(-t/\tau)] d(\ln \tau).$ 

<sup>b</sup>IUPAC recommends *molar mass* (MM), which has SI units of g/mol. But *molecular weight* (MW) is widely used, and ACS accepts both terms. However, MW is in fact a dimensionless ratio that is numerically very close to MM (g/mol), and one cannot "change its units." The number often called "molecular weight (kg/mol)" is actually MW/1000 (no units). This quantity can properly be called *molar mass* with units of kg/mol.  ${}^{c}J(t) = \int_{-\infty}^{\infty} L(\tau)[1 - \exp(-t/\tau)]d(\ln \tau) + t/\eta_0.$ 

<sup>d</sup>Because there is not a true plateau in G(t) or  $G'(\omega)$ ,  $G_N^0$  is inferred from  $G'(\omega)$  and  $G''(\omega)$  using methods reviewed by Liu *et al.* [Polymer **47**, 4461–4479 (2006)].



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Name	Definition	Symbol	SI units
Vertical shift factor <sup>a</sup>	$b_T G'(T, \omega a_T) = G'(T_0, \omega)$	$b_T$	
Horizontal shift factor	$b_T G(T, \omega a_T) = G(T_0, \omega)$ $-c_1 (T - T_0)$	$a_T$	
First WLF coefficient	$\log_{10}a_T = \frac{c_1(T - T_0)}{c_2 + (T - T_0)}$	$c_1$	—
Second WLF coefficient	$\log_{10}a_T = \frac{-c_1(T - T_0)}{c_2 + (T - T_0)}$	<i>c</i> <sub>2</sub>	К
Activation energy for flow	$a_T(T) = \exp\left[\frac{E_a}{R}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$	$E_{\mathrm{a}}$	kJ/mol

<sup>a</sup>The vertical shift factor  $b_T$  should be  $T_0\rho_0/T\rho$ , where  $\rho$  is mass density,  $\rho_0 = \rho(T_0)$ , or  $b_T$  should be set equal to one. It should not be used as a fitting parameter.

Name	Definition	Symbol	SI units
Stress relaxation (step strain)			
Strain amplitude		γo	_
Relaxation modulus (nonlinear)	$\sigma(t)/\gamma_0$	$G(t, \gamma_0)$	Pa
Damping function in shear	$G(t, \gamma_0)/G(t)$	$h(\gamma_0)$	_
First normal stress relaxation function		$N_1(t, \gamma_0)$	Ра
Second normal stress relaxation function		$N_2(t, \gamma_0)$	Ра
First normal stress relaxation coefficient	$N_1(t,\gamma_0)/\gamma_0^2$	$\Psi_1(t,\gamma_0)$	Pa
Second normal stress relaxation coefficient	$N_2(t,\gamma_0)/\gamma_0^2$	$\Psi_2(t,\gamma_0)$	Pa
Start-up shear (at fixed shear rate)			
Shear stress growth function		$\sigma^+(t,\dot{\gamma})$	Pa
Shear stress growth coefficient	$\sigma^+(t,\dot{\gamma})/\dot{\gamma}$	$\eta^+(t,\dot{\gamma})$	Pa s
First normal stress growth function	$\sigma_{11} - \sigma_{22}$	$N_1^+(t,\dot{\gamma})$	Ра
First normal stress growth coefficient	$N_1^+(t,\dot{\gamma})/\dot{\gamma}^2$	$\Psi_1^+(t,\dot{\gamma})$	Pa s <sup>2</sup>
Second normal stress growth function	$\sigma_{22} - \sigma_{33}$	$N_2^+(t,\dot{\gamma})$	Pa
Second normal stress growth coefficient	$N_{2}^{+}(t,\dot{y})/\dot{y}^{2}$	$\Psi_2^+(t,\dot{\gamma})$	Pa s <sup>2</sup>
Stress ratio	$N_1(\dot{\gamma})/\sigma(\dot{\gamma})$	2 ( ) • )	
Cessation of steady shear	$\dot{\gamma} = 0$ from $t = 0$		
Shear stress decay function		$\sigma^{-}(t,\dot{\gamma})$	Pa
Shear stress decay coefficient	$\sigma^{-}(t,\dot{\gamma})/\dot{\gamma}$	$\eta^{-}(t,\dot{\gamma})$	Pa s
First normal stress decay function	$\sigma_{11} - \sigma_{22}$	$N_1^-(t,\dot{\gamma})$	Pa
First normal stress decay coefficient	$N_1^-(t,\dot{\gamma})/\dot{\gamma}^2$	$\Psi_1^-(t,\dot{\gamma})$	Pa s <sup>2</sup>
Second normal stress decay function	$\sigma_{22} - \sigma_{33}$	$N_2^-(t,\dot{\gamma})$	Pa
Second normal stress decay coefficient	$N_2^-(t,\dot{\gamma})/\dot{\gamma}^2$	$\Psi_2^-(t,\dot{\gamma})$	Pa s <sup>2</sup>
Creep and creep recovery (recoil)			
Creep compliance	$\gamma(t,\sigma)/\sigma$	$J(t,\sigma)$	$Pa^{-1}$
Steady-state compliance <sup>a</sup>	$J(t ightarrow\infty,\sigma)$	$J_{ m s}(\sigma)$	$Pa^{-1}$
Recoverable strain (after $t_0$ when $\sigma \rightarrow 0$ )	$\gamma[t_0,\sigma] - \gamma[t,\sigma]  t > t_0$	$\gamma_{\mathbf{r}}(t',\sigma)  t' \equiv t - t_0$	—
Ultimate recoil	$\gamma_{\rm r}(t' \to \infty,  \sigma)$	$\gamma_\infty(\sigma)$	—
Steady-state recoverable compliance <sup>a</sup>	$\gamma_\infty(\sigma)/\sigma$	$J_{ m s}(\sigma)$	$Pa^{-1}$

#### TABLE IV. Nonlinear viscoelasticity in shear.

<sup>a</sup>Although measured in different ways, the steady-state compliance and the steady-state recoverable compliance should be equal to each other according to the Boltzmann principle.

Name	Definition	Symbol	SI units
Tensile (uniaxial) extension			
Engineering strain <sup>a</sup>	$(L - L_0)/L_0$	3	_
Engineering stress <sup>a</sup>	$F/A_0$	σ	Ра
Young's modulus of a solid	$\sigma/\varepsilon$	Ε	Ра
Net tensile stress (true) (see Fig. 2)	$\sigma_{ m zz}-\sigma_{rr}$	$\sigma_{ m E}$	Pa
Hencky strain of a liquid	$\ln(L/L_0)$	3	_
Hencky strain rate	dɛ/dt	ż	$s^{-1}$
Tensile stress growth function		$\sigma^+_{ m E}(t,\dotarepsilon)$	Ра
Tensile stress growth coefficient	$\sigma^+_{ m E}(t,\dot{arepsilon})/\dot{arepsilon}$	$\eta_{\rm E}^+(t,\dot{\varepsilon})$	Pa s
Extensional viscosity	$\eta_{\rm F}^+(t,\dot{\varepsilon})  (t\to\infty)$	$\eta_{\rm E}(\dot{\varepsilon})$	Pa s
Tensile creep compliance	$\epsilon(t)/\sigma_{\rm E}$	$D(t,\sigma)$	$Pa^{-1}$
Recoverable strain (after $t_0$ when $\sigma_E \rightarrow 0$ )	$\varepsilon[t_0,\sigma] - \varepsilon[t,\sigma]  t > t_0$	$\varepsilon_{\rm r}(t',\varepsilon)$ $t'\equiv t-t_0$	—
Biaxial extension (symmetrical)			
Biaxial strain	$\ln(R/R_0)$	$\varepsilon_{\mathrm{B}}$	_
Biaxial strain rate	$d(\ln R)/dt$	έ <sub>B</sub>	$s^{-1}$
Net biaxial stretching stress	$\sigma_{rr} - \sigma_{zz}$	$\sigma_{ m B}$	Ра
Biaxial stress growth function		$\sigma^+_{\rm B}(t,\dot{\varepsilon}_{\rm B})$	Ра
Biaxial stress growth coefficient	$\sigma^+_{ m B}(t,\dot{arepsilon}_{ m B})/\dot{arepsilon}_{ m B}$	$\eta_{\rm B}^+(t,\dot{\varepsilon}_{\rm B})$	Pa s
Biaxial stress decay coefficient	$\sigma_{\rm B}^{-}(t,\dot{\epsilon}_{\rm B})/\dot{\epsilon}_{\rm B}$	$\eta_{\rm B}^{-}(t,\dot{\varepsilon}_{\rm B})$	Pa s
Biaxial extensional viscosity	$\sigma^+_{\rm B}(t \to \infty, \dot{\varepsilon}_{\rm B})/\dot{\varepsilon}_{\rm B}$	$\eta_B(\dot{\varepsilon}_B)$	Pa s
Biaxial creep compliance	$\varepsilon_{\rm B}(t)/\sigma_{\rm B}$	$D(t,\sigma_{\rm B})$	$Pa^{-1}$

TABLE V. Nonlinear viscoelasticity in extension.

<sup>a</sup>In the mechanics literature, the same symbols are often used for both engineering and true stress and strain, but they are only equivalent in the limit of very small deformations.

### TABLE VI. Rheometry.

Name	Definition	Symbol	SI units
Capillary rheometers			
Apparent wall shear stress	$P_{\rm d}R/2L$	$\sigma_{ m A}$	Pa
Apparent wall shear rate	$4Q/\pi R^3$	Ϋ́Α	$s^{-1}$
Wall shear stress	$-\sigma_{rz} (r=R)$	$\sigma_{ m w}$	Pa
Wall shear rate	$-\mathrm{d}v/\mathrm{d}r$ ( $r=R$ )	Ϋ́w	$s^{-1}$
Cone-plate rheometers			
Cone angle	Figure 2	β	rad
Angular displacement	Figure 2	φ	rad
Angular velocity	$d\phi/dt$	Ω	rad/s
Torque	$2\pi R^3 \sigma_{\varphi\theta}/3^{a}$	М	N m
Normal thrust		Fz	Ν

<sup>a</sup>Approximation valid for  $\beta < 0.1$  rad.



FIG. 2. Symbols describing cone-plate geometry.

TABLE VII. Solutions.

Name	Definition	Symbol	SI units
Concentration		С	$\mathrm{kg}~\mathrm{m}^{-3}$ or $\mathrm{kg/m}^3$
Viscosity of solvent		$\eta_{ m s}$	Pa s
Relative viscosity	$(\eta/\eta_{\rm s})$	$\eta_{ m r}$	_
Specific viscosity	$(\eta_r - 1)$	$\eta_{ m sp}$	_
Reduced viscosity	$\eta_{\rm sp}/c$	$\eta_{\mathrm{red}}$	$m^3 kg^{-1}$ or $m^3/kg$
Intrinsic viscosity	$\lim_{\substack{\dot{\gamma} \to 0 \\ c \to 0}} \eta_{\rm red}$	$[\eta]$	$m^3 kg^{-1}$
Viscosity of matrix		$\eta_{ m m}$	Pa s



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Name	Definition	Symbol	SI units
Volume fraction solid	$V_{\rm solid}/V_{\rm suspension}$	$\phi$	_
Local stress tensor		<b>σ</b> (x,t)	Pa
Total bulk stress	$\Sigma^{ m f}+\Sigma^{ m p}$	Σ	Pa
Fluid contribution		$\Sigma^{\mathrm{f}}$	Pa
Particle contribution		$\Sigma^{\mathrm{p}}$	Pa
First normal stress difference	$\Sigma_{11} - \Sigma_{22}$	$N_1$	Pa
Second normal stress difference	$\Sigma_{22}-\Sigma_{33}$	$N_2$	Pa
Particle pressure	$-rac{1}{3}(\Sigma_{11}^{\mathrm{p}}+\Sigma_{22}^{\mathrm{p}}+\Sigma_{33}^{\mathrm{p}})$	П	Pa
Hydrodynamic particle stress	<u> </u>	$\Sigma^{\mathrm{H}}$	Pa
Interparticle stress		$\Sigma^{\mathrm{IP}}$	Pa
Brownian stress		$\Sigma^{\mathrm{B}}$	Pa
Fluid viscosity		$\eta_{ m f}$	Pa s
Effective viscosity of suspension	$\Sigma_{12}/\dot{\gamma}$	η	Pa s
Relative viscosity	$\eta/\eta_{\rm f}$	$\eta_{ m r}$	_
Particle contribution to $\eta_r$		$\eta_{ m P}$	Pa s
Maximum packing fraction <sup>a</sup>	(see note)	$\phi_{ m max}$	—

TABLE VIII. Suspensions (see also Péclet Number in Table XII).

<sup>a</sup>The maximum packing fraction for monodisperse spheres is thought of as the limiting value of the solid fraction for which shear flow can occur, implying that the effective viscosity diverges above this point. As a practical matter for measurement,  $\phi_{max}$  is a measure of a statically stable packing of the particles and is not a precisely known quantity. It varies from random loose packing,  $\phi_{rep} \approx 0.55$ , to random close packing,  $\phi_{rep} \approx 0.64$ ; the frictional properties at the particle surfaces may thus play a role in the observed packing. For polydisperse spheres,  $\phi_{max}$  is generally larger than for the monodisperse case.

Name	Definition	Symbol	SI units
Interfacial tension		$\sigma_{lphaeta}$	N/m = Pa m
Surface pressure	$\sigma_{lphaeta,0}-\sigma_{lphaeta}$	П	N/m = Pa m
Interfacial shear stress	$F^{\rm s}/L$	$\sigma^{ m s}$	Pa m
Interfacial shear strain	$dX_1/dx_2$	$\gamma^{s}$	_
Interfacial shear rate	$dv_1/dx_2$	γs	$s^{-1}$
Interfacial dilatational strain	$\ln(A/A_0)$	$\alpha^{s}$	
Interfacial dilatational strain rate	$d(\ln A)/dt$	$\dot{\alpha}^{s}$	$s^{-1}$
Steady shear and dilation			
Interfacial shear viscosity	$\sigma^{ m s}/\dot{\gamma}^{ m s}$	$\eta^{s}$	Pa s m
Interfacial dilatational viscosity	$\sigma^{\rm s}/\dot{lpha}^{\rm s}$	$\kappa^{s}$	Pa s m
Simple shear			
Interfacial shear modulus	$\sigma^{ m s}/\gamma^{ m s}$	$G^{\rm s}$	Pa m
Relaxation modulus (shear)	$\sigma^{\rm s}(t)/\gamma^{\rm s}$	$G^{\mathrm{s}}\left(t ight)$	Pa m
Pure dilation			
Interfacial dilatational modulus	$\sigma^{ m s}/lpha$	$K^{\rm s}$	Pa m
Dilatational storage modulus	$\sigma^{\rm s}(t)/\alpha$	$K^{\rm s}(t)$	Pa m
Gibbs elasticity (surfactants)	$d\sigma_{\alpha\beta}/d(\ln A)$	$E_{\Pi}$	Pa m
Small-amplitude oscillatory shear			
Strain amplitude	$\gamma^{\rm s} = \gamma^{\rm s}_0 \sin \omega t$	$\gamma_0^s$	—
Phase angle (loss angle)	$\sigma^{\rm s}(t) = \sigma^{\rm s}_0[\sin(\omega t + \delta)]$	δ	rad
Stress amplitude	(see "Interfacial shear stress" above)	$\sigma_0^{ m s}$	Pa m
Complex interfacial modulus	$G^{s\prime}+iG^{s\prime\prime}$	$G^{s_{*}}$	Pa m
Absolute magnitude of $G^{**}$	$\sigma_0^{\rm s}/\gamma_0^{\rm s}$	$ G^{s_{*}} $	Pa m
Storage modulus	$ G^{\mathrm{s}*} \cos\delta$	$G^{s\prime}$	Pa m
Loss modulus	$ G^{s*}  \sin \delta$	$G^{s_{\prime\prime}}$	Pa m
Complex viscosity	$\eta^{s\prime} - i\eta^{s\prime\prime}$	$\eta^{s*}$	Pa s m
Absolute magnitude of $\eta^{s*}$	$\sigma_0^{\rm s}/\omega\gamma_0^{\rm s}$	$ \eta^{s}* $	Pa s m
Dynamic viscosity (in phase with strain rate)	$G^{s\prime\prime}/\omega$	$\eta^{s'}$	Pa s m
Out-of-phase (with strain rate) component of $\eta^{s_{\ast}}$	$G^{s\prime}/\omega$	$\eta^{s\prime\prime}$	Pa s m
Small-amplitude oscillatory dilation			
Dilatational strain amplitude	$\alpha^{\rm s} = \alpha_0^{\rm s} \sin \omega t$		_
Complex dilatational modulus	$E^{s\prime} + iE^{s\prime\prime}$	$E^{s*}$	Pa m
Absolute magnitude of $E^*$	$\sigma_0^{ m s}/lpha_0^{ m s}$	$ E^{s}* $	Pa m
Storage dilatational modulus	$ E^{s_{*}} \cos \delta$	$E^{s\prime}$	Pa m
Loss dilatational modulus	$ E^* \sin \delta$	$E^{s\prime\prime}$	Pa m
Complex dilatational viscosity	$\kappa^{s\prime} - i\kappa^{s\prime\prime}$	$\kappa^{s*}$	Pa s m
Absolute magnitude of $\kappa^{s*}$	$\sigma_0^{\rm s}/\omega\delta_0^{\rm s}$	$ \kappa^{s}* $	Pa s m
Dynamic viscosity	$E^{s\prime\prime}/\omega$	$\kappa^{s\prime}$	Pa s m
Out-of-phase component of $\kappa^{s*}$	$E^{s\prime}/\omega$	$\kappa^{s\prime\prime}$	Pa s m
Other properties			
Creep compliance (shear)	$\gamma^{\rm s}(t)/\sigma^{\rm s}$	$J^{s}(t)$	$Pa^{-1} m^{-1}$
Equilibrium compliance of solid	$J^{\rm s}(t) \ (t \to \infty)$	$J_0^{\mathrm{e}}$	$Pa^{-1} m^{-1}$
Recoverable compliance	$J^{ m s}(t)-t/\eta_0^{ m s}$	$J_{\rm r}^{\rm s}(t)$	$\mathrm{Pa}^{-1} \mathrm{m}^{-1}$
Steady-state compliance of fluid	$J^{\rm s}(t) - t/\eta_0^{\rm s}$ $(t \to \infty)$	$J_0^{s}$	$Pa^{-1} m^{-1}$
Extensional viscosity	$\eta^{ m s}_{ m E}(t,\dot{arepsilon}^{ m s})~~(t o\infty)$	$\eta_{\rm E}^{\rm s}(\dot{\varepsilon}^{\rm s})$	Pa s m

TABLE IX. Interfacial and surface rheology (see also Boussinesq Number in Table XII).

a	tube diameter; average entanglement spacing/mesh size $(\sqrt{\langle R^2 \rangle_0 M_e/M})$
$b_{\rm K}$	Kuhn segment length <sup>a</sup>
f	tension in a chain segment
$f_{\max}$	maximum tension in a chain segment
k <sub>B</sub>	Boltzmann's constant, $1.38 \times 10^{-23}$ J/K
L	mean tube contour length
М	molecular weight, dimensionless <sup>b</sup>
М	molar mass g mol <sup>-1b</sup>
Me	molecular weight between entanglements <sup>c</sup> ( $\rho RT/G_{\rm N}^0$ )
$N_{\rm K}$	number of Kuhn segments in equivalent freely jointed chain <sup>a</sup>
N <sub>A</sub>	Avogradro's number, $6.023 \times 10^{23}$ molecules/mole
р	packing length <sup>d</sup> $(M/[\langle R^2 \rangle_0 \rho N_A])$
R	end-to-end distance of polymer molecule
R <sub>max</sub>	fully extended chain length <sup>a</sup>
S	tube contour variable (curvilinear coordinate along tube)
S	tube orientation tensor
Ζ	number of entanglements per molecule $(M/M_e)$
Greek letters	
α	dilution exponent for $M_e$
ζ	friction coefficient
ζο	monomer friction coefficient
λ	chain stretch; stretch ratio
ξ	correlation length; characteristic size scale (blob size)
$ au_{ m d}$	reptation (tube disengagement) time
$ au_{ m e}$	Rouse time of an entanglement strand $(\tau_R/Z^2)$
$ au_{ m p}$	relaxation time of the <i>p</i> th mode ( <i>p</i> is the mode index)
$\tau_{\rm R}$	Rouse stress relaxation time $(\zeta N^2 b^2)/(6\pi^2 kT)^e$

TABLE X.	Molecular	description	n of entangled	polymers.
			6	

<sup>a</sup> $b_{\rm K}$  and  $N_{\rm K}$  are defined by the following relationships:  $\langle R^2 \rangle = b_{\rm K}^2 N_{\rm K} R_{\rm max} = b_{\rm K} N_{\rm K}$ .

<sup>b</sup>IUPAC recommends *molar mass* (MM), which has SI units of g/mol. But *molecular weight* (MW) is widely used, and ACS accepts both terms. However, MW is in fact a dimensionless ratio that is numerically very close to MM (g/mol), and one cannot "change its units." The number often called "molecular weight (kg/mol)" is actually MW/1000 (no units). This quantity can properly be called *molar mass* with units of kg/mol.

<sup>c</sup>This is the definition originally proposed by John Ferry. The following alternative definition was introduced much later, but its use can be confusing, and it should not be used:  $M_e \equiv \frac{4}{5}\rho RT/G_N^0$ .

<sup>d</sup>For a discussion of *p*, see Fetters *et al.* [Macromolecules **27**, 4639 (1994)].

<sup>e</sup>For linear molecules, Doi and Edwards call  $\tau_s$  the Rouse rotational relaxation time, for which they use the symbol  $\tau_r$ , which is conjectured to be equal to  $2\tau_R$ .

Total stress tensor $\sigma$ Extra stress tensor $\tau$ Strain tensor for linear viscoelasticity $\gamma$ Cauchy tensor $C$ Finger tensor $B$ or $C^{-1}$ Doi-Edwards strain tensor $Q$ Rate-of-strain tensor <sup>ai</sup> $\dot{\gamma} = \nabla v + \nabla v^{\mathrm{T}}$		
Extra stress tensor $\tau$ Strain tensor for linear viscoelasticity $\gamma$ Cauchy tensor $C$ Finger tensor $B$ or $C^{-1}$ Doi-Edwards strain tensor $Q$ Rate-of-strain tensor <sup>a</sup> $\dot{\gamma} = \nabla v + \nabla v^{\mathrm{T}}$	Total stress tensor	σ
Strain tensor for linear viscoelasticity $\gamma$ Cauchy tensor $C$ Finger tensor $B$ or $C^{-1}$ Doi-Edwards strain tensor $Q$ Rate-of-strain tensor <sup>a</sup> $\dot{\gamma} = \nabla v + \nabla v^{\mathrm{T}}$	Extra stress tensor	τ
Cauchy tensorCFinger tensorB or $C^{-1}$ Doi-Edwards strain tensorQRate-of-strain tensor <sup>a</sup> $\dot{\gamma} = \nabla v + \nabla v^{\mathrm{T}}$	Strain tensor for linear viscoelasticity	γ
Finger tensor $B$ or $C^{-1}$ Doi-Edwards strain tensor $Q$ Rate-of-strain tensor <sup>a</sup> $\dot{\gamma} = \nabla v + \nabla v^{\mathrm{T}}$	Cauchy tensor	С
Doi-Edwards strain tensor $Q$ Rate-of-strain tensor <sup>a</sup> $\dot{\gamma} = \nabla v + \nabla v^{\mathrm{T}}$	Finger tensor	<b>B</b> or $C^{-1}$
Rate-of-strain tensor <sup>a</sup> $\dot{\gamma} = \nabla v + \nabla v^{\mathrm{T}}$	Doi-Edwards strain tensor	Q
	Rate-of-strain tensor <sup>a</sup>	$\dot{\gamma} =  abla v +  abla v^{ ext{T}}$

**TABLE XI.** Stress and strain tensors.

<sup>a</sup>An alternative definition, equal to  $\frac{1}{2}\dot{\gamma}$ , is widely used in fluid mechanics and is acceptable, but the symbol **D** should be used for this tensor to avoid confusion:  $\mathbf{D} \equiv \frac{1}{2} (\nabla v + \nabla v^T)$ .

Deborah number <sup>a</sup>	De (characteristic time of fluid)/(duration of deformation)
Weissenberg number <sup>a,b</sup>	Wi (characteristic time of fluid) × (rate of deformation) = e.g., $\tau \dot{\epsilon}$ or $\tau \dot{\gamma}$
Boussinesq number	Bo (surface shear stress)/[(bulk subphase shear stress) $\times$
	(perimeter length along which the surface shear stress acts)]
Péclet number <sup>c</sup>	$Pe \equiv \dot{\gamma} a^2 / D_o$ (a = particle radius; $D_o$ = particle diffusion coefficient)

**TABLE XII.** Dimensionless groups used to describe experimental regimes.

<sup>a</sup>The definitions and uses of these groups are explained in detail in "Weissenberg and Deborah Numbers—Their definition and use," Rheol. Bull. (The Society of Rheology) **79**(2), 14 (2010).

<sup>b</sup>The Weissenberg number has sometimes been considered to be  $(N_1/\sigma)$ . However, this is a ratio of dependent rather than independent variables and thus describes data rather than experimental conditions. The quantity  $(N_1/\sigma)$  is often called the *stress ratio*.

<sup>c</sup>Characteristic rate of advection over the rate of diffusion for suspensions in shear flow.